Int. J. Heat Mass Transfer. Vol. 7, pp. 95-100. Pergamon Press 1964. Printed in Great Britain.

SIMPLIFIED CALCULATION FOR COOLER CONDENSERS FOR GAS-MULTICOMPONENT VAPOUR MIXTURES

TOKURO MIZUSHINA, HIROYUKI UEDA,* SHINICHIRO IKENO† and KO ISHII
Department of Chemical Engineering, Kyoto University, Kyoto, Japan

(*Received* 24 *July* 1963)

Abstract—"Three points method", a simplified calculation for cooler condensers is applied to the system of gas-multicomponent vapour. It is shown that the results of calculation are in agreement with the experimental values with a satisfactory accuracy.

NOMENCLATURE

A,	area of transfer surface, [m^2];
C,	specific heat, [kcal/kg degC];
G,	mass velocity of gas vapour mixture, [kg/m^2 h];
h,	coefficient of heat transfer, [kcal/m^2 h degC];
h_0	combined conductance other than the gas film [kcal/m^2 h degC];
j,	j factor in heat transfer;
k,	thermal conductivity, [kcal/m h degC];
k_g,	mass-transfer coefficient, [kg-mole/m^2 h];
M,	molecular weight;
P,	total pressure in condenser, [mm Hg];
p,	partial vapour pressure, [mm Hg];
Q,	heat transferred, [kcal/h];
t,	temperature, [°C];
$\Delta t = t_i - t_l$,	[degC];
λ,	latent heat of vaporization, [kcal/kg-mole];
Pr,	Prandtl number;
Sc,	Schmidt number.

Subscripts

1,	inlet of gas;
2,	outlet of gas;
av,	average value;
c,	condensate film;
g,	gas main stream or gas film;
i,	interface;
l,	cooling water;
m,	middle point.

INTRODUCTION

SINCE Colburn and Hougen [1] published a stepwise calculation method for the thermal design of cooler condensers, a large number of methods have been proposed by various investigators [2–7]. Recently Mizushina *et al.* proposed "Three points method", a simplified calculation which takes a short time to perform and gives fairly accurate results.

All of these methods are applicable to gas mixtures which contain only one kind of vapour. Though the graphical method may be extended to multicomponent systems as suggested by Mizushina *et al.* [8] and Porter and Jeffreys [9], it is erroneous and terribly tedious.

In this paper "Three points method" will be extended to multicomponent systems.

1. PROCESS OF CALCULATION

(a) The gas rate, the vapour concentrations and the gas temperature at inlet, the percentage of moles of vapour to be condensed and the temperature of cooling water available may be given.

(b) Assuming that the outlet gas mixtures are in equilibrium with the liquid of the concentration of the vapour component ratio in the inlet gas mixture, and calculating the total vapour pressure there from the values in (a), the temperature

---

* Present address: Kawasaki Dockyard Co. Ltd.

† Present address: Mitsui Chemical Co. Ltd.

and the partial pressure of each vapour component are determined. Since this is not the actual case, the values thus determined may be erroneous. The discrepancies of the calculated values from the measured ones are shown in the example calculation. These, however, do not cause the serious error in the final results on the heat transfer surface area.

(c) From the total heat balance, the total heat transferred and the rate and the outlet temperature of the cooling water are determined. In this balance, the temperature of the condensate is assumed to go down to that of cooling water for the sake of conservative calculation.

(d) A fictitious middle point where the gas temperature, the rate of each component of vapour, and the cooling water temperature are the arithmetic mean of those at gas inlet and outlet is assumed.

(e) The heat balances of Colburn–Hougen type as shown below are applied to the three points, i.e. inlet, middle and outlet to calculate the temperatures at the interface between condensate film and gas at those points.

$$h_o(t_i - t_l) = h_g(t_g - t_i) + \lambda k_g \ln \frac{P - \sum p_i}{P - \sum p_g}. \qquad (1)$$

The average value of the combined conductance h_o can be calculated from the heat-transfer coefficient of cooling water and the vapour film coefficient computed from Nusselt's equation. This average value is used for the all three points.

The gas film coefficient of heat transfer, h_g, and that of mass transfer, k_g, are calculated from the following equations:

$$h_g = \frac{j C_g G}{(Pr)^{1/2}} \qquad (2)^*$$

$$k_g = \frac{j G}{M_g (Sc)^{1/2}}. \qquad (3)^*$$

In these equations the average values of Pr and Sc are used for all the three points.

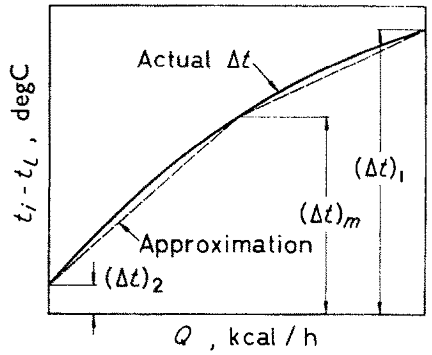

FIG. 1. $(t_i - t_l)$ vs. Q.

(f) As shown in Fig. 1 the relation of $\Delta t = t_i - t_l$ and Q is assumed to be represented by two straight lines—the dotted lines—between the inlet and the middle point and between the middle and the outlet point respectively, though the actual temperature difference is supposed to be as the solid line curve. Hence, the heat-transfer surface area can be calculated by

$$A = \frac{Q/2}{h_o\left\{[(\Delta t)_1 - (\Delta t)_m] \Big/ \left[2{\cdot}3 \log \frac{(\Delta t)_1}{(\Delta t)_m}\right]\right\}} + \frac{Q/2}{h_o\left\{[(\Delta t)_m - (\Delta t)_2] \Big/ \left[2{\cdot}3 \log \frac{(\Delta t)_m}{(\Delta t)_2}\right]\right\}}. \qquad (4)$$

On the other hand, assuming that the curve of $1/\Delta t$ vs. Q is parabolic, so called Simpson's method was applied to calculate the surface area, but the error was larger than that of (4).

2. EXPERIMENTS

The experimental apparatus is a vertical single tube cooler condenser the same as described in a previous paper [7]. In the inside copper tube of 21·9 mm i.d., 28·6 mm o.d. and 0·128 m² in effective area, the gas mixture flows, and in the annular space between the inside tube and outside polyvinyl chloride tube of 51·5 mm i.d., the cooling water flows countercurrently to gas flow. The systems studied are, air–methanol–water, air–methanol–benzene, air–benzene–water, air–toluene–water, and air–acetone–methanol–water. The data are listed in Tables 1 and 2.

---

* In the case of gas system [10], the power to Prandtl and Schmidt modulus is 1/2 rather than 2/3.

Table 1. *Results of experiment and calculation. Two vapour components*

System	Run No.	Gas mixture at inlet					Cooling water		Vapour to be condensed	Results of calculation	Error	Results of Simpson's method	Error
		Rate [kg/h]	p_{air} [mm Hg]	p_{II} [mm Hg]	p_{III} [mm Hg]	t_{e_1} [degC]	Rate [kg/h]	t_{l_2} [degC]	[mole per cent]	A[m²]	[per cent]	A[m²]	[per cent]
air–methanol–water	1	19·12	198·3	481·6	80·1	79·7	2040	19·9	68·5	0·144	+12·5	0·143	+11·7
	2	17·33	150·7	514·5	94·8	79·5	2040	18·7	72·9	0·140	+ 9·4	0·139	+ 8·6
	3	11·59	320·5	362·7	76·8	83·2	2030	17·0	71·6	0·131	+ 2·3	0·135	+ 5·5
	4	18·01	105·5	537·4	117·1	82·0	2030	18·1	77·8	0·155	+21·1	0·152	+18·8
	5	16·83	200·5	313·3	246·2	87·0	2030	15·4	77·9	0·138	+ 7·8	0·139	+ 8·6
	6	14·66	119·2	353·0	287·8	83·8	2020	15·4	86·7	0·152	+18·8	0·163	+27·3
	7	11·01	318·8	235·4	205·8	88·9	1850	16·5	77·9	0·140	+ 9·4	0·149	+16·4
	8	15·18	166·6	303·2	290·2	87·6	1770	15·9	82·0	0·149	+16·4	0·151	+18·0
air–methanol–benzene	11	15·80	336·1	324·1	99·8	75·0	2020	9·5	69·8	0·147	+14·8	0·148	+15·6
	12	15·28	259·9	354·8	145·3	70·0	2020	8·9	70·5	0·123	− 3·9	0·124	− 3·1
	13	12·86	190·5	441·5	128·0	77·3	2020	9·7	83·2	0·148	+15·6	0·158	+23·4
	14	16·69	333·2	300·6	126·2	81·8	2020	9·8	68·7	0·147	+14·8	0·149	+16·4
	15	15·04	270·1	346·4	143·5	76·8	2020	9·5	74·2	0·148	+15·6	0·151	+18·0
	16	17·40	334·6	259·9	165·5	72·1	2010	8·8	69·9	0·149	+16·4	0·151	+18·0
	17	16·50	263·2	304·4	192·4	75·5	2010	8·8	75·3	0·146	+14·1	0·149	+16·4
	18	17·71	341·5	227·5	191·0	75·6	2010	8·9	67·5	0·142	+10·9	0·143	+11·7
	19	15·63	205·0	297·3	257·7	76·8	2010	8·9	79·3	0·141	+10·2	0·145	+13·3
air–benzene–water*	21	22·03	207·9	392·5	159·6	72·7	1320	29·0	64·0	0·142	+ 9·2	0·140	+ 7·7
air–toluene–water*	31	18·06	222·2	239·0	298·8	83·5	1210	11·6	87·2	0·173	+33·1	0·190	+46·2

* In these cases the surface area of the experimental apparatus is 0·13 m². The large error of Run No. 31 may come from the two phase condensate film of this case.

Table 2. *Result of experiment and calculation. Three vapour components*

System	Run No.	Gas mixture at inlet						Cooling water		Vapour to be condensed	Results of calculation	Error	Results of Simpson's method	Error
		Rate [kg/h]	p_{air} [mm Hg]	p_{I} [mm Hg]	p_{III} [mm Hg]	p_{IV} [mm Hg]	t_{e_1} [degC]	Rate [kg/h]	t_{l_2} [degC]	[mole per cent]	A[m²]	[per cent]	A[m²]	[per cent
air–acetone–methanol–water	41	13·82	347·9	134·7	180·5	96·9	71·8	1970	3·1	71·5	0·139	+ 8·6	0·142	+10·9
	42	11·28	244·2	162·4	232·7	120·7	75·1	1980	3·8	80·9	0·154	+20·3	0·164	+28·1
	43	12·21	354·8	65·5	241·4	98·3	83·3	1985	5·6	75·5	0·144	+11·3	0·148	+15·6
	44	9·88	262·3	68·9	312·0	116·8	86·6	1985	7·2	80·3	0·150	+17·2	0·163	+27·3
	45	10·39	271·6	157·8	187·1	143·5	87·6	1990	5·6	80·6	0·149	+16·4	0·161	+25·8
	46	11·87	370·7	84·2	173·0	132·1	89·4	1985	6·7	74·1	0·135	+ 5·5	0·142	+10·9

3. EXAMPLE OF CALCULATION

Run No. 17 of the experiments will be taken as an example.

(a) *Problem*

Gas and vapour rate at inlet:

air;	3·89 kg/h	p_{A1}	= 263·2 mm Hg
methanol;	4·95 kg/h	p_{M1}	= 304·4 mm Hg
benzene;	7·66 kg/h	p_{B1}	= 192·4 mm Hg
total;	16·50 kg/h	P	= 760 mm Hg

Gas temperature at inlet: $t_{g_1} = 75{\cdot}5$°C.

Vapour to be condensed: 75·3 mole %.

Cooling water rate: 2010 kg/h.

Cooling water temperature: $t_{l_2} = 8{\cdot}8$°C.

(b) *Gas outlet condition*

Assumed concentration of liquid contacting the gas at outlet

$$= \frac{304{\cdot}4}{304{\cdot}4 + 192{\cdot}4} = 0{\cdot}612 \text{ methanol.}$$

The vapour pressure in equilibrium with the liquid:

methanol;	p_{M2}	= 161·4 mm Hg
benzene;	p_{B2}	= 80·3 mm Hg.

Hence,

air;	3·89 kg/h
methanol;	1·34 kg/h
benzene;	1·62 kg/h.

The temperature of gas: $t_{g_2} = 38{\cdot}0$°C.

For comparison, the measured values at outlet will be shown:

$p_{M2} = 135{\cdot}3$ mm Hg

$p_{B2} = 105{\cdot}4$ mm Hg

$t_{g_2} = 43{\cdot}8$°C.

These discrepancies will not cause serious error in the final result on the heat-transfer surface area.

(c) *Total heat balance*

air; $(3{\cdot}89)(0{\cdot}24)(75{\cdot}5 - 38{\cdot}0) = 35$ kcal/h

methanol; $(1{\cdot}34)(0{\cdot}4)(75{\cdot}5 - 38{\cdot}0) + (4{\cdot}95 - 1{\cdot}34) \times [(0{\cdot}4)(75{\cdot}5 - 64{\cdot}7) + 260 + (0{\cdot}612)(64{\cdot}7 - 8{\cdot}8)] = 1099$ kcal/h.

benzene; $(1{\cdot}62)(0{\cdot}28)(75{\cdot}5 - 38{\cdot}0) + (7{\cdot}66 - 1{\cdot}62) \times [(0{\cdot}28)(75{\cdot}5 - 8{\cdot}8) + 106{\cdot}2] = 768$ kcal/h

$\therefore Q = 35 + 1099 + 768 = 1902$ kcal/h.

Cooling water outlet temperature;

$$t_{l1} = \frac{1902}{(2010)(1)} + 8{\cdot}8 = 9{\cdot}7\text{°C}.$$

(d) *Condition at the middle point*

$$t_{g_m} = \frac{75{\cdot}5 + 38{\cdot}0}{2} = 56{\cdot}8\text{°C}.$$

Rate of air = 3·89 kg/h.

$$\text{Rate of methanol} = \frac{4{\cdot}95 + 1{\cdot}34}{2} = 3{\cdot}15 \text{ kg/h}$$

$$\text{Rate of benzene} = \frac{7{\cdot}66 + 1{\cdot}62}{2} = 4{\cdot}64 \text{ kg/h}$$

$$p_{Am} = \frac{(760)(3{\cdot}89/29)}{\dfrac{3{\cdot}89}{29} + \dfrac{3{\cdot}15}{32} + \dfrac{4{\cdot}64}{78}} = 415{\cdot}9 \text{ mm Hg}$$

$$p_{Mm} = \frac{(760)(3{\cdot}15/32)}{\dfrac{3{\cdot}89}{29} + \dfrac{3{\cdot}15}{32} + \dfrac{4{\cdot}64}{78}} = 214{\cdot}3 \text{ mm Hg}$$

$$p_{Bm} = \frac{(760)(4{\cdot}64/78)}{\dfrac{3{\cdot}89}{29} + \dfrac{3{\cdot}15}{32} + \dfrac{4{\cdot}64}{78}} = 154{\cdot}6 \text{ mm Hg}$$

$$t_{l_m} = \frac{8{\cdot}8 + 9{\cdot}7}{2} = 9{\cdot}3\text{°C}.$$

(e) *Interface temperature*

Combined conductance h_o:

water film coefficient was obtained from the preliminary experiment. In a design calculation, this value should be obtained from one of heat transfer equations:

$$h_l = 1580 \text{ kcal/m}^2 \text{ h degC.}$$

Vapour film coefficient can be calculated with Nusselt's equation from the following values:

$$\rho_{cm} = 843\ \mathrm{kg/m^3}$$

$$\mu_{cm} = 0{\cdot}530\ \mathrm{c.p.} = 1{\cdot}91\ \mathrm{kg/m\ h}$$

$$k_{cm} = 0{\cdot}144\ \mathrm{kcal/m\ h\ degC}$$

$$\Gamma = \frac{(4{\cdot}95 + 7{\cdot}66) - (1{\cdot}34 + 1{\cdot}62)}{(0{\cdot}0219)(3{\cdot}14)} = 140\ \mathrm{kg/m\ h}$$

$$\therefore\ h_c = 950\ \mathrm{kcal/m^2\ h\ degC}$$

$$\therefore\ h_o = \frac{1}{950} + \frac{0{\cdot}0219}{0{\cdot}0286}\left(\frac{1}{1580}\right) = 650\ \mathrm{kcal/m^2\ h\ degC.}$$

Gas film coefficients h_g and k_g: at middle point

$$C_{g_m} = 0{\cdot}299\ \mathrm{kcal/kg\ degC}, \quad M_{g_m} = 40.$$

Hence, from Eucken's equation

$$(Pr)_{av} = \frac{0{\cdot}299}{0{\cdot}299 + \dfrac{2{\cdot}48}{40}} = 0{\cdot}83.$$

The binary diffusion coefficients for air–methanol and air–benzene can be calculated from one of the equations for estimation of them as

$$(\mathscr{D}_{AM})_m = 0{\cdot}190\ \mathrm{cm^2/s} = 0{\cdot}0684\ \mathrm{m^2/h}$$

$$(\mathscr{D}_{AB})_m = 0{\cdot}116\ \mathrm{cm^2/s} = 0{\cdot}0418\ \mathrm{m^2/h.}$$

Approximately $(\mathscr{D}_{Am})_m$ may be computed from

$$(\mathscr{D}_{AM})_m = x_{c2}\frac{1}{(\mathscr{D}_{AM})_m} + (1 - x_{c2})\frac{1}{(\mathscr{D}_{AB})_m}$$

where x_{c2} = mole fraction of methanol of the condensate at outlet = 0·595

$$(\mathscr{D}_{Am})_m = 0{\cdot}0544\ \mathrm{m^2/h.}$$

Again, at middle point

$$\rho_{g_m} = 1{\cdot}478\ \mathrm{kg/m^3}$$

$$\mu_{g_m} = 0{\cdot}0459\ \mathrm{kg/m\ h.}$$

Hence

$$(Sc)_{av} = \frac{0{\cdot}0459}{(1{\cdot}478)(0{\cdot}0544)} = 0{\cdot}571.$$

Then, at inlet point

$$G = \frac{16{\cdot}50}{(\pi/4)(0{\cdot}0219)^2} = 43830\ \mathrm{kg/m^2\ h}$$

$$\mu_g = 0{\cdot}0486\ \mathrm{kg/m\ h}$$

$$Re = \frac{(0{\cdot}0219)(43830)}{0{\cdot}0486} = 19750.$$

The preliminary experiment gives

$$j = 0{\cdot}0248\,(Re)^{-0{\cdot}2} \qquad \therefore j = 0{\cdot}00343.$$

And $C_g = 0{\cdot}317$ kcal/kg degC, $M_g = 42{\cdot}6$

Putting these values into (2) and (3)

$$h_g = \frac{(0{\cdot}00343)(0{\cdot}317)(43830)}{(0{\cdot}83)^{1/2}} = 52{\cdot}5\ \mathrm{kcal/m^2\ h\ degC}$$

$$k_g = \frac{(0{\cdot}00343)(43830)}{(42{\cdot}6)(0{\cdot}571)^{1/2}} = 4{\cdot}68\ \mathrm{kg\text{-}mole/m^2\ h.}$$

Average value of molal heat of vaporization:

$$\lambda_{av} = 7930\ \mathrm{kcal/kg\text{-}mole.}$$

Thus, (1) becomes

$$650\,(t_i - 9{\cdot}7) = 52{\cdot}5\,(75{\cdot}5 - t_i) + 7930 \times 4{\cdot}68 \times 2{\cdot}3 \log\frac{760 - \sum pi}{760 - 496{\cdot}8}$$

$$\therefore\quad t_i = 43{\cdot}3\,^{\circ}\mathrm{C}$$

$$\therefore\ (\Delta t)_1 = 43{\cdot}3 - 9{\cdot}7 = 33{\cdot}6\ \mathrm{degC.}$$

Applying the same process of calculation as shown above to the middle and outlet point, the following results are obtained.

At middle point

$$Re = 14800$$

$$h_g = 37{\cdot}1\ \mathrm{kcal/m^2\ h\ degC}$$

$$k_g = 3{\cdot}74\ \mathrm{kg\text{-}mole/m^2\ h}$$

$$t_i = 32{\cdot}9\,^{\circ}\mathrm{C}$$

$$(\Delta t)_m = 32{\cdot}9 - 9{\cdot}3 = 23{\cdot}6\ \mathrm{degC.}$$

At outlet point

$$Re = 9230$$

$$h_g = 21{\cdot}9\ \mathrm{kcal/m^2\ h\ degC}$$

$$k_g = 2{\cdot}77\ \mathrm{kg\text{-}mole/m^2\ h}$$

$$t_i = 18{\cdot}1\,^{\circ}\mathrm{C}$$

$$(\Delta t)_2 = 18{\cdot}1 - 8{\cdot}8 = 9{\cdot}3\ \mathrm{degC}.$$

(f) *Heat-transfer surface area*

From (4)

$$A = \frac{(1902)(2{\cdot}3)}{(2)(650)}\left[\frac{\log(33{\cdot}6/23{\cdot}6)}{33{\cdot}6 - 23{\cdot}6} + \frac{\log(23{\cdot}6/9{\cdot}3)}{23{\cdot}6 - 9{\cdot}3}\right] = 0{\cdot}146\ \mathrm{m}^2$$

$$\text{error} = \left(\frac{0{\cdot}146 - 0{\cdot}128}{0{\cdot}128}\right)(100) = +\,14{\cdot}1\ \text{per cent.}$$

On the other hand, Simpson's calculation gives

$$A = \left(\frac{1902}{650}\right)\left(\frac{1}{6}\right)\left(\frac{1}{33{\cdot}6} + \frac{4}{23{\cdot}6} + \frac{1}{9{\cdot}3}\right) = 0{\cdot}149\ \mathrm{m}^2$$

$$\text{error} = \left(\frac{0{\cdot}149 - 0{\cdot}128}{0{\cdot}128}\right)(100) = +\,16{\cdot}4\ \text{per cent.}$$

4. RESULTS OF CALCULATION

The results of calculation for the transfer surface area are listed in Tables 1 and 2, which are in fairly good agreement with the experimental value. And the results of (4) seem to be better than those of Simpson's method.

5. CONCLUSION

The simplified method, i.e. "Three points method" can be applied to the calculation of cooler condensers for gas-multicomponent vapour system and gives a satisfactory accuracy, not so high as the case of one vapour component (error for which is within 10 per cent) but still good enough for the design purpose in a short time computation. Instead of the middle point, 1/3 and 2/3 point may be assumed. This "Four points method" may be better than "Three points method".

ACKNOWLEDGEMENTS

The authors wish to express their thanks to the Asahi Glass Co. Ltd. for their financial support for this research.

REFERENCES

1. A. P. Colburn and O. A. Hougen, Design of cooler condensers of mixtures of vapors with noncondensing gases, *Industr. Engng. Chem.* **26**, 1178–1182 (1934).
2. G. H. Bras, Design of cooler condensers for vapor-gas mixtures; *Chem. Engng.* **60**, 223–226, 238–240 (1953).
3. G. H. P. Bras, A graphical method for the calculation of cooler-condensers, *Chem. Engng. Sci.* **6**, 277–282 (1957).
4. G. H. P. Bras, The graphical determination of changes in the gas phase due to simultaneous heat and mass transfer, *Chem. Engng. Sci.* **9**, 176–181 (1958).
5. B. Huldén, Condensation of vapours from gas-vapour mixtures. An approximate method of design, *Chem. Engng. Sci.* **7**, 60–65 (1957).
6. T. Mizushina, N. Hashimoto and M. Nakajima, Design of cooler condensers for gas–vapour mixtures, *Chem. Engng. Sci.* **9**, 195–204 (1959).
7. T. Mizushina, M. Nakajima and T. Oshima, Study on cooler condensers for gas–vapour mixtures, *Chem. Engng. Sci.* **13**, 7–17 (1960).
8. T. Mizushina, J. Oishi and N. Hashimoto, Method of calculations for the absorption accompanied with heat liberation, *Chem. Engng. Sci.* **10**, 31–36 (1959).
9. T. Mizushina and M. Nakajima, Simultaneous heat and mass transfer. The relation between heat transfer and material transfer coefficients, *Kagaku-Kogaku* **15**, 30–34 (1951).
10. K. E. Porter and G. V. Jeffreys, The design of cooler-condensers for the condensation of binary vapours in the presence of a non-condensible gas, *Trans. Instn. Chem. Engrs.* **41**, 126–139(1963).

Résumé—"La méthode des trois points" qui simplifie le calcul des condenseurs refroidisseurs est appliquée aux systèmes de vapeur à plusieurs composants gazeux. On montre que les résultats du calcul sont en accord avec les valeurs expérimentales suivant une précision satisfaisante.

Zusammenfassung—Die „Drei-Punkt-Methode", ein vereinfachtes Verfahren zur Berechnung von Kühler-Kondensatoren, wird auf das System Gas-Mehrkomponentendampf angewendet. Die Ergebnisse der Rechnung stimmen befriedigend mit den Versuchsresultaten überein.

Аннотация—«Метод трёх точек»—упрощённый способ расчёта конденсаторов применяется для систем газ-многокомпонентный пар. Показано, что результаты вычисления с достаточной точностью согласуются с экспериментальными величинами.